Dispersion Relations for Waves propagating in Composite Fermion Gases

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Abstract

The discrete Uehling-Uhlenbeck equations are solved to study the propagation of plane (sound) waves in a system of composite fermionic particles with hard-sphere interactions and the filling factor (ν) being 1/2. The Uehling-Uhlenbeck collision sum, as it is highly nonlinear, is linearized firstly and then decomposed by using the plane wave assumption. We compare the dispersion relations thus obtained by the relevant Pauli-blocking parameter B which describes the different-statistics particles for the quantum analog of the discrete Boltzmann system when B is positive (Bose gases), zero (Boltzmann gases), and negative (Fermi Gases). We found, as the effective magnetic field being zero (ν=1/2 using the composite fermion formulation), the electric and fluctuating (induced) magnetic fields effect will induce anomalous dispersion relations.

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1 Introduction

The study of the electronic properties of quasi-two-dimensional (2D) systems has resulted in a number of remarkable discoveries in the past two decades [1-6]. Among the most interesting of these are the integral and fractional quantum Hall effects [1] (the integral quantum Hall effect, which is manifested by the development of spectacularly flat plateaus in the Hall conductance centered at integral values of n, was discovered in 1980 by Klaus von Klitzing). In both of these effects, incompressible states of a 2D electron liquid are found at particular values of the electron density for a given value of the magnetic field applied normal to the 2D layer.

In the presence of a strong magnetic field B transverse to a two-dimensional system of electrons, the tiny cyclotron orbits of an electron are quantized to produce discrete kinetic energy levels, called *Landau levels*. The degeneracy of each Landau level-that is to say, its maximum population per unit area-is $\mathcal{B}/\phi_0$, where $\phi_0 = h/e$ is the elementary quantum of magnetic flux. This degeneracy implies that the number of occupied Landau levels, called the filling factor, is $\nu = \rho\phi_0$ $/\mathcal{B}$, where ρ is the two-dimensional electron density.

The fractional quantum Hall effect (FQHE) is more difficult to understand and more interesting in terms of new basic physics. The energy gap that gives rise to the Laughlin [2] incompressible fluid state is completely the result of the interaction between the electrons. The elementary excitations are fractionally charged Laughlin quasiparticles, which satisfy fractional statistics [2]. The standard techniques of many-body perturbation theory are incapable of treating FQH

systems because of the complete degeneracy of the single-particle levels in the absence of the interactions. Laughlin [2] was able to determine the form of the ground-state wavefunction and of the elementary excitations on the basis of physical insight into the nature of the many-body correlations. Striking confirmation of Laughlins picture was obtained by exact diagonalization of the interaction Hamiltonian within the subspace of the lowest Landau level of small systems [2]. Jain, Lopez and Fradkin,and Halperin *et al.* [3-4] have extended Laughlins approach and developed a composite-fermion (CF) description of the 2D electron gas in a strong magnetic field. The composite-fermion (CF) picture offers a simple intuitive way of understanding many of the surprising properties of a strongly interacting two-dimensional electron fluid in a large magnetic field.

The quickest way to introduce the composite fermion is through the following series of steps, which Jain called the *Bohr theory* of composite fermions because it obtains some of the essential results with the help of an oversimpified but useful picture [7]. The outcome is that strongly interacting electrons in a strong magnetic field $\mathcal{B}$ transform into weakly interacting composite fermions in a weaker effective magnetic field $\mathcal{B}_{\text{eff}}$, given by $\mathcal{B}_{\text{eff}} = \mathcal{B} - 2p\,\phi_0\rho$, where $2p$ is an even integer. Equivalently, one can say that electrons at filling factor n convert into composite fermions with filling factor $\nu^* = \rho\phi_0/|\mathcal{B}_{\text{eff}}|$, given by

$$\nu = \frac{\nu^*}{2p\,\nu^* \pm 1}.$$

The minus sign corresponds to situations when $\mathcal{B}_{\text{eff}}$ points antiparallel to $\mathcal{B}$. Start by considering interacting electrons in the transverse magnetic field $\mathcal{B}$. Now attach to each electron an infinitely thin, massless magnetic solenoid carrying 2p flux quanta pointing antiparallel to $\mathcal{B}$, turning it into a composite fermion. Such a conversion preserves the minus sign associated with an exchange of two fermions, because the bound state of an electron and an even number of flux quanta is itself a fermion. Hence the name. It also leaves the Aharonov-Bohm phase factors associated with all closed paths unchanged, because the additional phase factor due to a flux $\phi = 2p\phi_0$ is $\exp[2i\pi\phi/\phi_0] = 1$. In other words, the attached flux is unobservable, and the new problem, formulated in terms of composite fermions, is identical to the one with which we began. The crucial point is that the many-particle ground state of electrons at $\nu < 1$ was highly degenerate in the absence of interaction, with all lowest Landau level configurations having the same energy. But now, the degeneracy of the composite-fermion ground state at the corresponding $\nu^* > 1$ is drastically smaller, even when the interaction between composite fermions is switched off. For integral values of ν^*, in fact, one gets a non-degenerate ground state. The reduced degeneracy suggests that one might start by treating the composite fermions as independent. In that approximation, the composite fermions fill a Fermi sea of their own whenever $\mathcal{B}_{\text{eff}}$ vanishes ($1/\nu = 2p$), and form composite-fermion Landau levels when it does not.

Composite fermions (CF), consisting of an electron with two flux quanta attached, provide a different approach to the fractional quantum Hall effect (FQHE) [3]. At filling factor $\nu = 1/2$ the attached flux quanta are 'compensated' by externally applied magnetic flux such that the CF move in a vanishing effective magnetic field. Away from $\nu = 1/2$ the effective magnetic field increases, the CF move on circles with radius $R_{C;CF}$ and the Landau quantization of the

circular motion of the new particles is the origin of the FQHE. The radius $R_{C;CF}$ is given by $\hbar\sqrt{4\pi n_s}/e\,\mathcal{B}_{\text{eff}}$ with the electron density n_s, the effective magnetic field $\mathcal{B}_{\text{eff}} = \mathcal{B} - \mathcal{B}_{1/2}$, and $\mathcal{B}_{1/2}$ the magnetic field at $\nu = 1/2$. Experimental evidence for the existence of CF mainly stems from commensurability experiments where the Fermi wave vector $k_{F;CF}$ of the novel quasi-particles is probed by a periodic external perturbation.

An important application of the concept concerns the metallic state at $\nu = 1/2$, where no fractional quantum Hall state is seen. If composite fermions exist at that filling factor, they would experience no effective magnetic field ($\mathcal{B}_{\text{eff}} = 0$). Thus a mean-field picture suggests a Fermi sea of composite fermions.

Although this CF description has offered a simple picture for the interpretation of many experimental results. However, the underlying reason for the validity of many of the approximations used with the CF approach is not completely understood [7-8].

1.1 Previous Semi-Classical Approaches

A semi-classical theory based on the Boltzmann transport equation for a two-dimensional electron gas modulated along one direction with weak electrostatic or magnetic modulations have been proposed [9-11]. Ustinov and Kravtsov studied the giant magnetoresistance effect in magnetic superlattices for the current perpendicular to and in the layer planes within a unified semiclassical approach that is based on the Boltzmann equation with exact boundary conditions for the spin-dependent distribution functions of electrons. Interface processes responsible for the magnetoresistance were found to be different in these geometries, and that can result in an essential difference in general behaviour between the in-plane magnetoresistance and the perpendicular-plane one. A correlation between the giant magnetoresistance and the multilayer magnetization is also discussed therein [9].

Boltzmann's equation provides an adequate starting point of transport calculations for two-dimensional electron systems in the presence of periodic electric and magnetic modulation fields, both in the regime of the low-field positive magnetoresistance and of the Weiss oscillations at intermediate values of the applied magnetic field. For example, Zwerschke and Gerhardts solved Boltzmann's equation by the method of characteristics, which allows to exploit explicitly information about the structure of the phase space. That structure becomes very complicated if the amplitudes of the modulation fields become so large and the average magnetic field becomes so small that, in addition to the drifting cyclotron orbits, channeled orbits exist and drifting cyclotron orbits extend over many periods of the modulation [10].

In Refs. [5,10], they considered the 2DEG in the x-y plane as a degenerate Fermi gas, with Fermi energy $E_F = m^* v_F^2/2$, of (non-interacting) particles with effective mass m^* and charge $-e$ obeying classical dynamics, i.e. Newton's equation $m^*\dot{\mathbf{v}} = -e[\mathbf{F} + (\mathbf{v} \times \mathcal{B})]$. In equilibrium, the electric field is given by $\mathbf{F}(\mathbf{r}) = \nabla V(\mathbf{r})/e$, where $V(\mathbf{r})$ is the modulating electrostatic potential. In thermal equilibrium, all states with energy below E_F are occupied, and for the linear response to an external homogeneous electric field $\mathbf{E}_0$ only the electrons with energy $E(\mathbf{r}, \mathbf{v}) = (m^* \mathbf{v}^2)/2 + V(\mathbf{r}) = E_F$ contribute to the current. The distribution function $f(\mathbf{r}, \mathbf{v}, t)$

obeys the Boltzmann equation

$$\frac{\partial f}{\partial t} + \mathcal{D} f - \mathcal{C}[f;\mathbf{r},\mathbf{v}] = \mathbf{v}\cdot\mathbf{E}_0,$$

where the drift term $\mathcal{D}$ describes the change due to the natural motion of the electrons in the modulation field (in absence of $\mathbf{E}_0$), and $\mathcal{C}$ is the collision operator. We might use polar coordinates in the velocity space, $\mathbf{v} = v\mathbf{u}$ with $v(\mathbf{r}) = v_F[1 - V(\mathbf{r})/E_F]^{1/2}$ and $\mathbf{u}(\Theta)$=$(\cos\Theta, \sin\Theta)$. Sometimes [2], the drift term reads $\mathcal{D} = \mathbf{v}\cdot\nabla + [\omega_c + \omega_{el}(\mathbf{r},\Theta)]\partial/\partial\Theta$, with cyclotron frequency $\omega_C = e\mathcal{B}_{\rm eff}/m^*$ and $\omega_{el}(\mathbf{r},\Theta) = (\nabla V)\mathbf{t}$ with $\mathbf{t}(\Theta) = (\sin\Theta, \cos\Theta)$.
Recently Jobst investigated the magnetoresistance of a weakly density modulated high mobility two-dimensional electron system around filling factor $\nu = 1/2$ [12]. The experimental ρ_{xx}-traces around $\nu = 1/2$ were well described by novel model calculations, based on a semiclassical solution of the Boltzmann equation, taking into account anisotropic scattering. We also noticed that, the effects of a tunable periodic density modulation imposed upon a 2D electron system have been probed using surface acoustic waves by Willett *et al.* [13]. A substantial effect was induced at filling factor 1/2 in which the Fermi surface properties of the CF are anisotropically replaced by features similar to those seen in quantum Hall states. The response measured using different SAW wavelengths and similarities in the temperature dependence between the modulation induced features at 1/2 and quantum Hall states were described therein [13].

1.2 Present Objectives

Motivated by the interesting issues about ν=1/2, we like to study their characteristics relevant to the sound propagation in CF gases here using our verified quantum (discrete) kinetic approaches [14-15]. In the discrete kinetic model approach [16], the main idea is to consider that the particle velocities belong to a given finite set of velocity vectors, e.g., $\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_p$, p is a finite positive integer. Only the velocity space is discretized, the space and time variables are continuous [15-16] (please see the detailed references therein). By using the discrete velocity model approach, the velocity of propagation of plane waves can be classically determined by looking for the properties of the solution of the conservation equation referred to the equilibrium state.
As a continuous attempt of ultrasonic propagation (in dilute Boltzmann gases [17]), considering the quantum analog of the discrete velocity model and the Uehling-Uhlenbeck collision term which could describe the collision of a gas of dilute hard-sphere Fermi-, Boltzmann- or Bose-particles by tuning a parameter θ [14,18] (via a *blocking factor* of the form $1+\theta f$ with f being a normalized distribution function giving the number of particles per cell, say, a unit cell, in phase space), in this paper, we plan to investigate the dispersion relations of plane ultrasonic waves propagating in composite-fermion gases by the quantum discrete kinetic model which has been verified before. The CF-CF interactions [6,8] will not be considered in present works since our present approach works quite well only for the dilute (weakly-interacting) regime [15-20]. This presentation will give more clues to the studies of the quantum wave dynamics in composite fermion gases [13].

2 Mathematical Formulations

The gas is presumed to be composed of identical hard-sphere particles of the same mass. The discrete number density (of particles) is denoted by $N_i(\mathbf{x}, t)$ associated with the velocity $\mathbf{u}_i$ at point $\mathbf{x}$ and time t. Following the CF model, around $\nu = 1/2$ or any even-denominator $\nu = 1/2p$, $2p$ fictitious magnetic flux quanta ($\phi_0 = h/e$) are attached to each electron in the direction opposite to the external magnetic field $\mathcal{B}$. The so formed composite particles follow Fermi statistics and are named composite fermions. The flux attachment transforms the strongly interacting two-dimensional electron system (2DES) of density ρ in a high a magnetic field into an equivalent weakly interacting CF system, which experiences a smaller effective magnetic field, $\mathcal{B}_{\text{eff}} = \mathcal{B} - 2\rho p\phi_0$. In particular, at exact even-denominator fillings, $\nu = 1/2p$, $\mathcal{B} = 2p\,\rho\,h/e = 2\rho p\,\phi_0$ and $\mathcal{B}_{\text{eff}}$ vanishes. Under these conditions, the CFs reside in a magnetic field-free region and, like ordinary 2D electrons at $\mathcal{B} = 0$, they form a Fermi sea.

If only nonlinear binary collisions and the effective magnetic field $\mathbf{B}_{\text{eff}}$ being zero (for $\nu = 1/2$ in the CF sense) are considered, we have for the evolution of N_i,

$$\frac{\partial N_i}{\partial t} + \mathbf{u}_i \cdot \nabla N_i - \frac{e(\mathbf{E} + \mathbf{u}_i \times \mathbf{B}_{\text{eff}})}{m^*} \cdot \nabla_{\mathbf{u}_i} N_i = \mathcal{C}_i \equiv \sum_{j=1}^{p} \sum_{(k,l)} (A_{kl}^{ij} N_k N_l - A_{ij}^{kl} N_i N_j), \quad i = 1, \cdots, p, \tag{1}$$

where $\mathbf{E}$ is the electric field, m^* is the effective mass of the particle, (k, l) are admissible sets of collisions [14-18]. We may also define the right-hand-side of above equation as

$$\mathcal{C}_i(N) = \frac{1}{2} \sum_{j,k,l} (A_{kl}^{ij} N_k N_l - A_{ij}^{kl} N_i N_j), \tag{2}$$

with $i \in \Lambda = \{1, \cdots, p\}$, and the summation is taken over all $j, k, l \in \Lambda$, where A_{kl}^{ij} are nonnegative constants satisfying [14-18] (i) $A_{kl}^{ji} = A_{kl}^{ij} = A_{lk}^{ij}$: *indistinguishability of the particles in collision*, (ii) $A_{kl}^{ij}(u_i + u_j - u_k - u_l) = 0$: *conservation of momentum in the collision*, (iii) $A_{kl}^{ij} = A_{ij}^{kl}$: *microreversibility condition*. The conditions defined for discrete velocities above are valid for elastic binary collisions such that momentum and energy are preserved. The collision operator is now simply obtained by joining A_{ij}^{kl} to the corresponding transition probability densities a_{ij}^{kl} through $A_{ij}^{kl} = S|\mathbf{u}_i - \mathbf{u}_j|\, a_{ij}^{kl}$, where,

$$a_{ij}^{kl} \geq 0, \qquad \sum_{k,l=1}^{p} a_{ij}^{kl} = 1, \quad \forall\, i, j = 1, \cdots, p;$$

with S being the effective collisional cross-section [14-18]. If all n ($p = 2n$) outputs are assumed to be equally probable, then $a_{ij}^{kl} = 1/n$ for all k and l, otherwise $a_{ij}^{kl} = 0$. Collisions which satisfy the conservation and reversibility conditions which have been stated above are defined an *admissible collision* [14-18].

With the introducing of the Uehling-Uhlenbeck collision term [18] in Eq. (1) or Eq. (2),

$$\mathcal{C}_i = \sum_{j,k,l} A_{kl}^{ij} \left[N_k N_l (1 + \theta N_i)(1 + \theta N_j) - N_i N_j (1 + \theta N_k)(1 + \theta N_l)\right], \tag{3}$$

for $\theta < 0$ we obtain a gas of Fermi-particles; for $\theta > 0$ we obtain a gas of Bose-particles, and for $\theta = 0$ we obtain Eq. (1).

From Eq. (3), the model of discrete quantum Boltzmann equation for dilute hard-sphere gases proposed in [18] is then a system of $2n(=p)$ semilinear partial differential equations of the hyperbolic type :

$$\frac{\partial}{\partial t}N_i+\mathbf{v}_i\cdot\frac{\partial}{\partial \mathbf{x}}N_i-\frac{e(\mathbf{E}+\mathbf{v}_i\times\mathbf{B}_{\mathrm{eff}})}{m^*}\cdot\nabla_{\mathbf{v}_i}N_i=\frac{cS}{n}\sum_{j=1}^{2n}N_jN_{j+n}(1+\theta N_{j+1})(1+\theta N_{j+n+1})$$

$$-2cSN_iN_{i+n}(1+\theta N_{i+1})(1+\theta N_{i+n+1}), \tag{4}$$

where $N_i=N_{i+2n}$ are unknown functions, and $\mathbf{v}_i=c(\cos[(i-1)\pi/n],\sin[(i-1)\pi/n])$, $i=1,\cdots,2n$; c is a reference velocity modulus [14-18]. The admissible collisions as $n=2$ are $(\mathbf{v}_1,\mathbf{v}_3)\longleftrightarrow(\mathbf{v}_2,\mathbf{v}_4)$.

We notice that the right-hand-side of the Eq. (4) is highly nonlinear and complicated for a direct analysis. As passage of the sound wave causes a small departure from an equilibrium resulting in energy loss owing to internal friction and heat conduction, we linearize above equations around a uniform equilibrium state (N_0) by setting $N_i(t,x)=N_0\ (1+P_i(t,x))$, where P_i is a small perturbation. The equilibrium here is presumed to be the same as in Refs. [14-15,18](in the absence of applied fields, the electrons will be at equilibrium and the distribution function will be the equilibrium distribution function $N_0(\epsilon-\mu_0)=[1+\exp(\epsilon-\mu_0)/k_B T)]^{-1}$, where μ_0 is the chemical potential, k_B is the Boltzmann constant, the corresponding Fermi surface is defined by the equations $\epsilon(\mathbf{k})=\mu_0$ in the quasi-momentum space, $\mathbf{k}$ is the wave vector). After some similar manipulations as mentioned in Refs. [15,17], with $B=\theta N_0$ [14-15], which gives or defines the (proportional) contribution from dilute Bose gases (if $\theta>0$, e.g., $\theta=1$), or dilute Fermi gases (if $\theta<0$, e.g., $\theta=-1$), we then have

$$[\frac{\partial^2}{\partial t^2}+c^2\cos^2\frac{(m-1)\pi}{n}\frac{\partial^2}{\partial x^2}+4cSN_0(1+B)\frac{\partial}{\partial t}]D_m-\frac{4cSN_0(1+B)}{n}\sum_{k=1}^{n}\frac{\partial}{\partial t}D_k=\mathrm{RHS}, \tag{5}$$

where $D_m=(P_m+P_{m+n})/2$, $m=1,\cdots,n$, since $D_1=D_m$ for $1=m\ (\mathrm{mod}\ 2n)$. Here, RHS denotes the contribution from the electric field and the fluctuating induced magnetic field. This term could be worked out by following the previous approaches [9,21] (cf. the second term in the left-hand side of the equation (4) in [9]).

We are ready to look for the solutions in the form of plane wave $D_m=d_m\exp i(kx-\omega t)$, $(m=1,\cdots,n)$, with $\omega=\omega(k)$. This is related to the dispersion relations of (forced) plane waves propagating in dilute (monatomic) hard-sphere Bose ($B>0$) or Fermi ($B<0$) gases. So we have

$$(1+ih(1+B)-2\lambda^2cos^2\frac{(m-1)\pi}{n})d_m-\frac{ih(1+B)}{n}\sum_{k=1}^{n}d_k=\mathrm{RHS},\qquad m=1,\cdots,n, \tag{6}$$

with

$$\lambda=kc/(\sqrt{2}\omega),\qquad h=4cSN_0/\omega,$$

where λ is complex and $h\,(\propto 1/\mathrm{Kn})$ is the rarefaction parameter of the Bose- or Fermi-particle gas (Kn is the Knudsen number which is defined as the ratio of the mean free path of Bose or Fermi gases to the wave length of the plane (sound) wave).

2.1 Weak External Fields

We firstly consider the case of rather weak electric field together with rather weak fluctuating (induced) magnetic field. It means RHS ≈ 0 considering other domainted terms in the equation (6). Let $d_m = \mathcal{C}/(1+ih(1+B)-2\lambda^2\cos^2[(m-1)\pi/n])$, where $\mathcal{C}$ is an arbitrary, unknown constant, since we here only have interest in the eigenvalues of above relation. The eigenvalue problems for different $2\times n$-velocity model reduces to

$$1-\frac{ih(1+B)}{n}\sum_{m=1}^{n}\frac{1}{1+ih(1+B)-2\lambda^2\cos^2\frac{(m-1)\pi}{n}} = \text{RHS} \sim 0. \tag{7}$$

We solve only $n=2$ case, i.e., 4-velocity case since for $n>2$ there might be spurious invariants [14-15]. For 2×2-velocity model, we obtain

$$1-[ih(1+B)/2]\sum_{m=1}^{2}\{1/[1+ih(1+B)-2\lambda^2\cos^2(m-1)\pi/2]\}=0.$$

3 Results and Discussions

With the filling factor ν=1/2, we are now ready to obtain the dispersion relations for sound propagating in composite fermion gases (with $\mathcal{B}_{\text{eff}}$=0) which might be useful to those subsequent studies reported in [13] by using surface acoustic waves. By using the standard symbolic or numerical software, e.g. Mathematica or Matlab, we can obtain the complex roots ($\lambda=\lambda_r+$ i λ_i) from the polynomial equation above. The roots are the values for the nondimensionalized dispersion (positive real part; a relative measure of the sound or phase speed) and the attenuation or absorption (positive imaginary part), respectively.

Curves in Fig. 1 or 2 follow the conventional dispersion relations of ultrasound propagation in dilute hard-sphere (Boltzmann; $B=0$) gases [17,19-20]. Here, s-scattering means the conventional s-wave scattering. Our results show that as $|B|$ (B: the Pauli-blocking parameter) increases, the dispersion (λ_r) will reach the continuum or hydrodynamical limit ($h\to\infty$) earlier. The phase speed of the plane (sound) wave in Bose gases (even for small but fixed h) increases more rapid than that of Fermi gases (w.r.t. to the standard conditions : $h\to\infty$) as the relevant parameter B increases. For all the rarefaction measure (h), plane waves propagate faster in Bose-particle gases than Boltzmann-particle and Fermi-particle gases. Meanwhile, the maximum absorption (or attenuation) for all the rarefaction parameters h keeps the same for all B as observed in Fig. 2. There are only shifts of the maximum absorption state (defined as h_{max}) w.r.t. the rarefaction parameter h when B increases. It seems for the same mean free path or mean collision frequency of the dilute hard-sphere gases (i.e. the same h as h is small enough but $h<h_{max}$) there will be more absorption in Bose particles than those of Boltzmann and Fermi particles when the plane (sound) wave propagates.

On the contrary, for the same h (as h is large enough but $h>h_{max}$, there will be less absorption in Bose particles than those of Boltzmann particles when the plane wave propagates. When B (i.e., θ) is less than zero or for the Fermi-particle gases, the resulting situations just mentioned above reverse. For instance, as the rarefaction parameter is around 10, which is near the hydrodynamical or continuum limit, we can observe that the ultrasound absorption becomes the

largest when the plane (sound) wave propagates in hard-sphere Fermi gases. That in Bose gases becomes the smallest. As also illustrated in Fig. 1 for cases of dilute Fermi gases ($B < 0$), the rather small dispersion value (relative measure of different phase speeds between the present rarefied state : h and the hydrodynamical state : $h \to \infty$) when B approaches to -1 perhaps means there is the Fermi pressure which causes a Fermi gas to resist compression.

From a modern point of view, dissipations of the (forced) plane (sound) wave arise fundamentally because of a necessary coupling between density and energy fluctuations induced by disturbances. Within one mean free path or so of an oscillating boundary, a free-particle flow solution can probably be computed. The damping will quite likely turn out to be linear because the damping mechanism is the shift in phase of particles which hit the wall at different times. As the wavelength is made significantly shorter, so that the effects of viscosity and the heat conduction are no longer small, the validity of hydrodynamic approach itself becomes questionable. If there is no rarefaction effect ($h = 0$), we have only real roots for all the models. Once $h \neq 0$, the imaginary part appears and the spectra diagram for each gas looks entirely different. In short, the dispersion ($k_r c/(\sqrt{2}\omega)$) reaches a continuum-value of 1 (or saturates) once h increases to infinity. We noticed that the increasing trend for the expression of our dispersion (λ_r; dimensionless) when waves propagating in Bose gases is similar to that (of dimensional sound speed) reported in Ref. [22-23]. The absorption or attenuation ($k_i c/(\sqrt{2}\omega)$) for our model, instead, firstly increases up to $h \sim 1$, depending upon the B values, then starts to decrease as h increases furthermore.

Although curves of the dispersion relation for hard-sphere Bose gases resemble qualitatively those reported in Refs. [22-23]. But, because of many unknown baselines (for example, in Fig. 1 of Ref. [22], their horizontal axis is represented by the condensate peak density which may be linked to our rarefaction parameter, however, at present, the detailed link is not available), we cannot directly compare ours with their data. The results presented here also show the intrinsic thermodynamic properties of the equilibrium states corresponding to the final equilibrium state after the collision of dilute hard-sphere Bose ($B > 0$), Boltzmann ($B = 0$), and Fermi ($B < 0$) gases.

At low temperatures, the Pauli exclusion principle forces Fermi-gas particles to be farther apart than the range of the collisional interaction, and they therefore cannot collide and rethermalize. That is to say, identical fermions are unable to undergo the collisions necessary to rethermalize the gas during evaporation because of the need to maximize Pauli blocking efects [15]. The much more spreading characteristics of dispersion relations for dilute Fermi gases ($B < 0$) obtained and illustrated in Figs. 1 and 2 seems to confirm above theoretical reasoning. The deviations in curves of dispersion and absorption shown in Figs. 1 and 2 also highlight their dissimilar quantum statistical nature.

Considering the case of nonzero electric and fluctuating (induced) magnetic fields, i.e., RHS $\neq 0$, we can obtain the detailed mathematical expression for RHS by following the verified approaches [9,21] with

$$\mathrm{RHS} \equiv i\frac{e(|\mathbf{E} + \mathbf{v}_i \times \mathcal{B}_{eff}|)}{m^*}\delta(\epsilon - \mu_0) c \cos(\frac{m-1}{n}\pi)[c\cos(\frac{m-1}{n}\pi)k + \omega], \tag{8}$$

where δ is the delta function. To obtain similar dispersion relations together with the equation (6) or (7) with nonzero RHS, we must impose the other condition from the equation (8) with RHS being zero for arbitrary $\mathcal{C}$. Under this situation, we have anomalous results : $|\lambda_r| = 1/\sqrt{2}$ (λ_r is negative!) and $\lambda_i = 0$ for all the rarefaction measure (hs) and the Pauli-blocking parameter (Bs). This strange behavior for ν=1/2 ($\mathcal{B}_{\text{eff}} = 0$, the electric field ($\mathbf{E}$) effect is being considered or both external fields are present) within the composite fermion formulation, however, is similar to that reported in [15] for the specific case of sound propagating in normal fermionic gases (the Pauli-blocking parameter $B = -1$) or sound propagating in dilute gases (for all Bs but with a free orientation parameter being $\pi/4$). There is no attenuation for above mentioned cases. This last observation might be relevant to the found *enhanced* conductivity (for 2D electron gases) corresponding to the even-denominator factor ν=1/2 (composite fermions) using surface acoustic waves (of wavelength smaller than 1 μm) [13] (geometric resonance of the composite fermions' cyclotron orbit and the ultrasound wavelength was also observed at smaller wavelength therein).

To conclude in brief, by using the quantum discrete kinetic approach, for the case of nonzero electric field, we obtain strange dispersion relations for waves propagating in CF gases with $\nu = 1/2$: $|\lambda_r| = 1/\sqrt{2}$ (λ_r is negative) and $\lambda_i = 0$ for all the rarefaction measure (hs) and the Pauli-blocking parameter (Bs). We shall investigate other interesting issues (e.g., compressibility of CF [24-25]) in the future. Acknowledgements. The author is partially supported by the Starting Funds for the 2005-XJU-Scholars.

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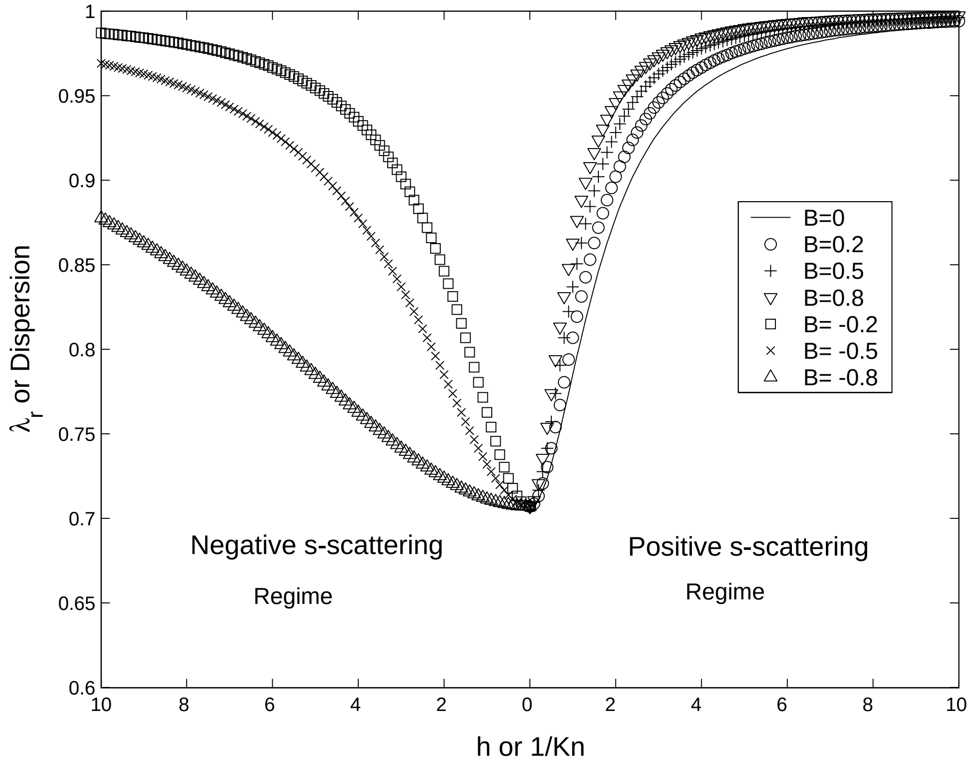

Fig. 1 Comparison of Bose- ($B > 0$), Boltzmann- ($B = 0$), and Fermi- ($B < 0$) particle effects on the dispersion (λ_r). s-scattering means the s-wave scattering. The electric field is rather weak and is neglected. The effective magnetic field $\mathcal{B}_{\text{eff}}$ is zero for $\nu = 1/2$ in CF sense. B is the Pauli-blocking parameter and is negative for the case of composite fermion gases.

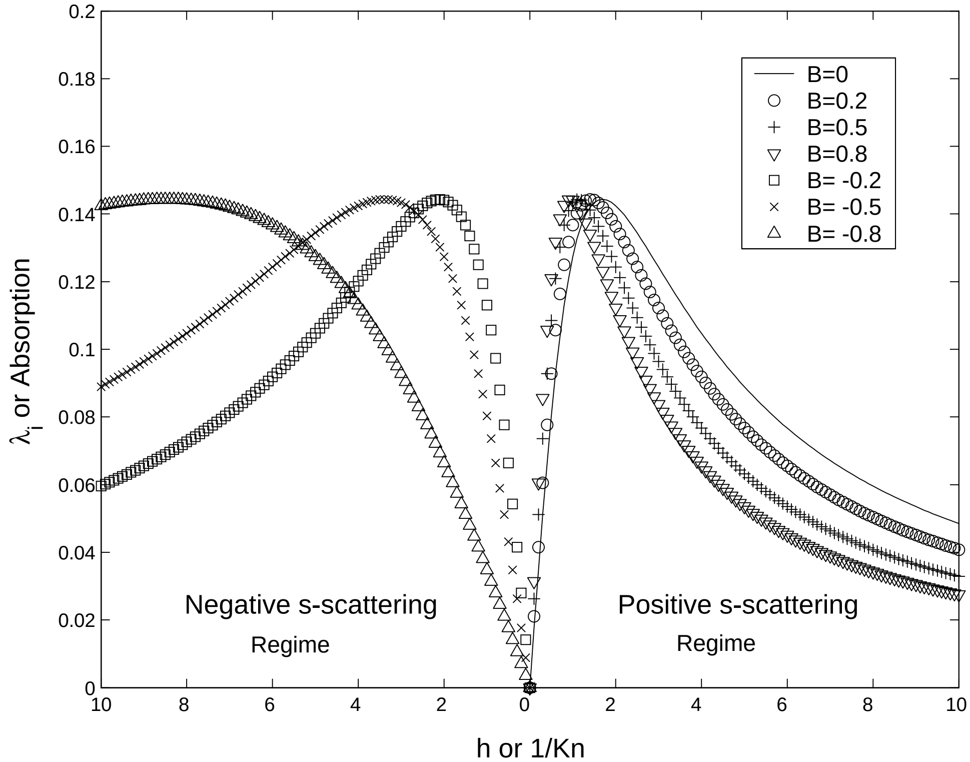

Fig. 2 Comparison of Bose- ($B > 0$), Boltzmann- ($B = 0$), and Fermi- ($B < 0$) particle effects on the absorption or attenuation (λ_i). The rarefaction measure $h = 4cSN_0/\omega$. The electric field is rather weak and is neglected. The effective magnetic field $\mathcal{B}_{\text{eff}}$ is zero for $\nu = 1/2$ in CF sense.